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Sticky Price Microfoundations in a Agent-Based Supply Chain

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Abstract I build a simple supply chain model with minimally rational agents and show how sticky prices are necessary to achieve equilibrium. Stickiness is necessary because prices take time to affect agents throughout the economy and changing prices too frequently leads to noise and disequilibrium. Agents are trial and error price makers. In single market economies they quickly achieve equilibrium through flexible prices. In supply chains they instead generate bullwhip dynamics without ever even orbiting around the correct prices and quantities. These agents are too simple to coordinate by centralizing information or any other standard supply-chain management strategy and are therefore forced to rely on prices alone. Price stickiness then becomes necessary and superior to flexible pricing and lets the economy achieve profit-maximizing equilibrium both in monopolist and competitive markets.

Keywords Agent-based models; Nominal rigidities; Supply chains; Disequilibrium; Microfoundations

1 Introduction

1.1 Motivation

As economists we see price flexibility as efficient and stickiness as an inferior compromise imposed by adjustment costs. I build a simple model with no adjustment costs where price stickiness is not only superior to flexible prices but necessary to achieve equilibrium. The model findings hinge on two main assumptions: delays to adapt to price changes and bounded rationality.

Consider an economy in disequilibrium where some goods are overproduced while others are in short supply. Prices have to change to incentivate agents to reallocate resources. What I focus on is how much time passes between the

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price changing and the agents reacting to it. The delay between a price changing and it having effect exists only with boundedly rational agents. Because all-knowing agents would predict any disequilibrium and adapt preemptively, a delay between a price changing and it having effect can only exist with bounded rationality.

Agents in this paper are trial and error price-makers. They quickly find equilibrium price and quantity when placed in a one-sector economy. When placed in a two-sector supply chain however, prices spiral out of control This is because firms downstream need time to adapt to a change in price upstream. This delay feeds into the trial and error of the upstream firms fooling them into thinking that prices are inelastic. To counter this inelasticity, upstream firms try ever larger price changes eventually overshooting and undershooting out of control. Sticky prices restore equilibrium by giving the time to agents to see their actions' full effect.

It is common to assume that prices can't be away from equilibrium for long. Traders would notice shortages or gluts and react by adjusting prices. For example in Mankiw's microeconomics textbook (Mankiw & Taylor, 2011) page 82 reads:

> 'Suppose first that the market price is above the equilibrium price [...] There is a surplus of the good: suppliers are unable to sell all they want at the going price. [...] They respond to the surplus by cutting their prices. Prices continue to fall until the market reaches the equilibrium.'

This kind of narrative is taken as a license to ignore disequilibrium altogether and assume market-clearing prices will emerge (Conlisk, 1996). I instead make explicit this adjustment process and show how it works well in simple markets but not in supply chains. Price-stickiness is required for equilibrium to emerge in supply chains. Rather than being a poor substitute for total flexibility, price stickiness is necessary for agents to deal with a slowly adapting world.

1.2 Research Contribution

Here I tie together two separate academic literatures. The first is the 'bullwhip effect': the large swings in prices we observe in supply chains that cannot be explained by changes in final demand (Baganha & Cohen, 1998). The second is 'sticky prices': the slowness in changing prices that we observe in macroeconomics in spite of evident changes in the final demand.

The operation research solution to bullwhip effects in supply chains is active management by centralizing information (Chen, Drezner, Ryan, & Simchi-Levi, 2000). The economy as a whole is a large supply chain, think input-output tables (Leontief, 1966), however it is so large and dispersed that centralized management is impossible. I show then that prices can coordinate the economy with no other information as long as they are sticky.

While my paper generates and explains bull-whip effects, the main thrust is on how to fix them when information is not available or cannot be processed.

For this reason I use simple agents with very limited rationality. It allows me to show how sticky prices are the method to coordinate supply chains and have them reach equilibrium. Sticky prices require little rationality and little information and are therefore perfect to manage the general supply chain that is the entire economy.

1.3 Roadmap

This paper is split into three main parts. The first part goes from the literature review in section 2 to section 4. In it I summarize and expand the Zero-Knowledge methodology. First in section 3 I show how agents can price their output through trial and error. I then show how, when there is a delay between price setting and demand adjusting to it, the trial and error rule oscillates away from equilibrium. Finally I show how price stickiness can recover the equilibrium. In section 4 I add production: agents can set their own production targets in order to maximize profits. Through marginal analysis agents are capable of reaching both monopolist and competitive equilibria.

The second part, section 5-6, deals with supply chains. In section 5 I plug the Zero-Knowledge traders into a supply chain. Because of the way the production targets downstream are set, the firms upstream face delayed demands. Again prices oscillate away from equilibrium unless prices are made sticky. In section 6 I go through various market structures for the supply chain and show how the results are robust to changes in market power.

The third part goes from section 7 to the conclusion. In this part of the paper I cast off some of the assumptions I made in the previous sections. I show how removing those assumptions create noisier results but the overall outcome is the same: supply chains keep on achieving equilibrium given sticky prices. I believe these sections are important as a robustness check of the previous results. In section 7 I let agents discover on their own if they are in a competitive or monopolist market. In section 8 I let agents set their own price stickiness. Finally in section 9 I show how one should structure empirical work around Zero-Knowledge traders.

The source code is available[1] on an open-source MIT license. The simulation is coded in Java and uses the MASON toolkit (Luke, Balan, Panait, Cioffi-Revilla, & Paus, 2003). I have personally replicated the one-market examples contained in this paper by recoding them in Dart[2] on MIT open-source license. Moreover the one-market examples were replicated semi-independently (I provided some clarifications and debugging but did no coding) as part of the ABCE modelling platform (Taghawi-Nejad, 2013) in Python[3]. Both of these replications do not restrict prices and quantities to be natural numbers like I do in this paper.

[1] As a git repository it is available here
[2] The repository is available here
[3] The repository is available here

2 Literature Review

There has always been a link between explicit price-making and price stickiness in the Keynesian tradition: Okun (1981) justified sticky prices in the Hicksian IS-LM model by claiming that 'in a world of price makers, rather than auctioneers and price takers, it takes time and resources to change prices'. Rather than assuming so however I here make this link explicit.

The agents in this paper are 'goal-oriented feedback mechanisms with learning', Pickering (1995)'s definition of 'cybernetics'.

My paper shares some similarities with the Beer-Distribution game (Sterman, 1995). Both deal with supply-chains, both have agents that act by feedback and both result in noise and disequilibrium. But the similarities end there. The fundamental difference is that my model has prices. In the beer distribution game, each node of the supply chain is powerless to influence the number of orders it receives, my firms can adjust their sale prices to throttle their customers' demand. Other components of the beer-distribution game are missing: here there is no exogenous shock to demand, no delay in clearing orders, no anchoring or 'wrongful mental simulation'(Sterman, 1989). The focus is entirely on prices and disequilibrium.

My paper's result also shares some superficial similarity with section 4.4 of 'Information Distortion in a Supply Chain' (Lee, Padmanabhan, & Whang, 2004). In it the authors describe the strategy of 'Every Day Low Price' where manufacturers reduce the frequency of discounts and promotions in order to stabilize supply chains. In both papers therefore rigid prices alleviate bullwhip effects. The causal mechanisms of the two papers are very different however. 'Every Day Low Price' keeps prices steady as a counter-measure to forward and strategic buying downstream that are incentivized by discounts and promotions. It is fundamentally a micro-economic result of managing other parties expectations. My paper has no forward or strategic buying and focuses instead on the steady prices coordinating low rationality agents in a low information environment.

The most extensive empirical review on sticky prices is Klenow and Malin (2010). The firm interviews by Blinder (1998) and Fabiani, Silvia et al. (2006) are both literature surveys on price stickiness microfoundations and empirical tests of which theory firms find more credible. Both highlight the importance of returning customers' goodwill (Okun, 1981), coordination between firms (Clower, 1965) and long-term contracts.

My paper is most similar to Blanchard (1982). In both papers price inertia is due to the desynchronization between firms in a supply-chain. While the results are similar the causality is reversed. There production adapts instantaneously but prices are set slowly and asynchronously which generates inertia within the supply chain. Here there is production inertia in the supply chain so that while prices can be set quickly it is counterproductive to do so.

In sticky-information models (Mankiw & Reis, 2002) the lack of knowledge alone can cause price-stickiness as information about shocks is expensive. The difference between my paper and the sticky-information literature is on how information gathering is modeled. In the sticky-information literature information gathering is a separate activity from trading: in the original Mankiw's paper a random proportion of firms receive information each day. In my paper information gathering is a by-product of trading. Firms set prices and see how many customers they attract; firms can only discover the demand by experimentation. Sticky prices become necessary when there is a large delay between running the experiment (setting a price) and seeing its result (changes in behavior along the supply-line).

3 Zero-Knowledge Traders and Delays

In this section I introduce the Zero-Knowledge traders and explain how they find the correct prices when selling a fixed daily inflow of goods. This trial and error mechanism is used for the rest of the paper. I then break the Zero-Knowledge pricing by adding arbitrary delays to how quickly the demand adapts to changes in prices. I finally show how sticky prices can solve the delays.

3.1 Trial and error pricing is effective when customers react immediately

Zero-Knowledge traders price their goods in a feedback loop. Every day a trader receives q^* goods to sell. In the morning, the trader sets sale price p and during the day it attracts q paying customers. If at the end of the day there are fewer customers than goods to sell, the trader will lower tomorrow's price. Defining the daily error as

$$e_t = q_t - q^*_t \tag{1a}$$
$$e_t = \text{Outflow} - \text{Inflow} \tag{1b}$$
$$e_t = \text{Netflow} \tag{1c}$$

The trader adjusts tomorrow prices through a PI(Proportional Integrative) controller rule:

$$p_{t+1} = ae_t + b\sum_{i=1}^{t} e_i + p_0 \tag{2}$$

Where p_0 is the initial random offset.

Start with a simple example: a price-maker agent receives 50 units of a good each day to sell, that is $q^*_t = 50, \forall t$. It faces a fixed but unknown daily demand curve $q_t = 101 - p_t$. Imagine that the agent starts with random initial price $p_0 = 80$. The first day the trader attracts $q_0 = 21$ customers and since its target was 50 sales its first error is $e_0 = -29$. The agent then plugs this error in the PI controller formula:

$$p_1 = a * (-29) + b * (-29) + 80$$

Assuming $a = b = 0.1$ the agent sets $p_1 = 74.2$. In this paper I allow only natural prices, so that p_1 is rounded to 74. The next day the seller attracts $q_1 = 27$. This generates the error $e_1 = -23$ which can be plugged in the PI controller to generate p_2:

$$p_2 = a * (-23) + b(-29 - 23) + 80$$

Figure 1 shows the simulated path generated by these parameters. The agent quickly finds the correct price.

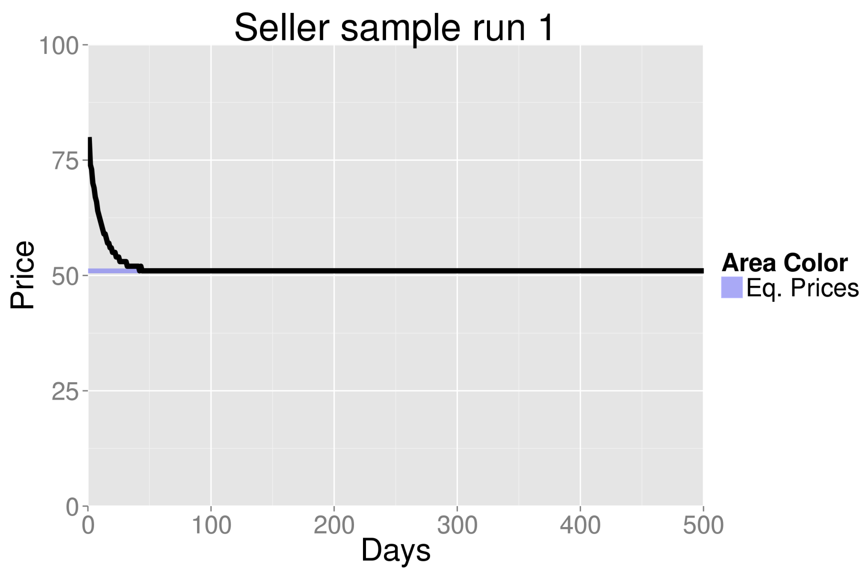

Figure 1: A sample run of a trader iteratively finding the correct prices when having 50 units of goods to sell and facing the daily linear demand $q_t = 101 - p_t$. This trader is using a PI controller with parameters $a = b = .1$.

3.2 Trial and error pricing fails if there is a long delay between setting a new price and it having effect

PI controllers simulate naíve trial and error pricing. As with all experimentation, PIs work better when trial results are informative and unambiguous. A simple way to mislead the agents is to add a time delay δ between a price p being set and the quantity demanded q adjusting to it.

Take a delayed demand curve, that is the quantity demanded at time t is a function of the price at time δ days before:

$$q_t = f(p_{t-\delta})$$

This delay is completely arbitrary and exogenous, I add it here to expose a weakness of adapting prices through PI controllers. A more endogenous source of delay is introduced in section 5.

Delays mean that even when the trader guesses the right price it takes δ days to yield the right quantity. The trader then will move away from the correct price because she thinks it is wrong. However the price is not wrong, it is the delay that is causing a mismatch between inflow and outflow. Depending on δ, the delay can slow down the approach to real prices (as in figure 2) or prevent it entirely (as in figure 3).

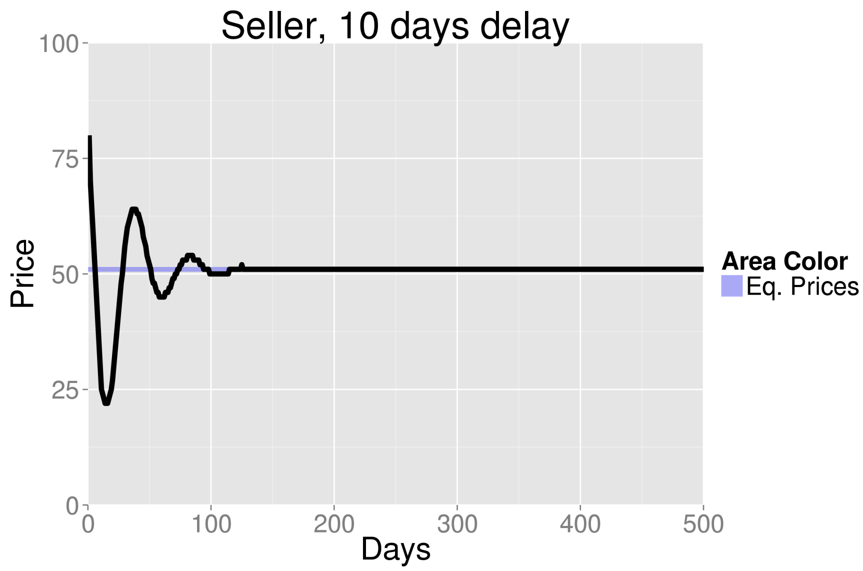

Figure 2: The same trader of Figure 1 now faces demand $q_t = 101 - p_{t-10}$, that is $\delta = 10$. The trader takes longer to find the right price.

3.3 Sticky prices are a solution to the price delay

The simplest way to deal with delays is to slow down the trial and error loop accordingly. If it takes a week for prices to have an effect, the trader can change prices every week rather than every day. Effectively, sticky prices. An agent

Figure 3: The same trader of figure 1 now faces demand $q_t = 101 - p_{t-20}$, that is $\delta = 20$. The trader never finds the right price.

using sticky prices continues to follow the same PI formula as equation 2 but does not activate it every day. Define the stickiness of an agent as s days, the agent activates its PI controller to change prices each day with probability $\frac{1}{s}$. In other words s is how many days on average pass between one change of price and another. This stochasticity is necessary to avoid creating spurious artifacts in simulations with multiple agents as they would otherwise proceed in lock-step.

Notice the significance of activating the PI controller only some days. The PI controller is there for the agent to adapt prices when they are perceived as wrong, that is when there is a non-zero error. By not activating the PI controller at any given day I am forcing the agent to maintain wrong prices. Why would it ever pay for an agent to keep an obviously wrong price? The answer is that the price is perceived as wrong today but it might not be wrong in the future. If the demand contains a delay, keeping prices constant allows the true demand q associated with current price p to emerge.

As shown in figure 4 adding stickiness $s = 20$ to an agent's pricing is enough to get back to equilibrium. Alternatively the trader can keep changing prices every day by smaller amounts so that the demand has time to catch up. This would mean using the PI equation 2 with small a and b. This also works as shown in figure 5

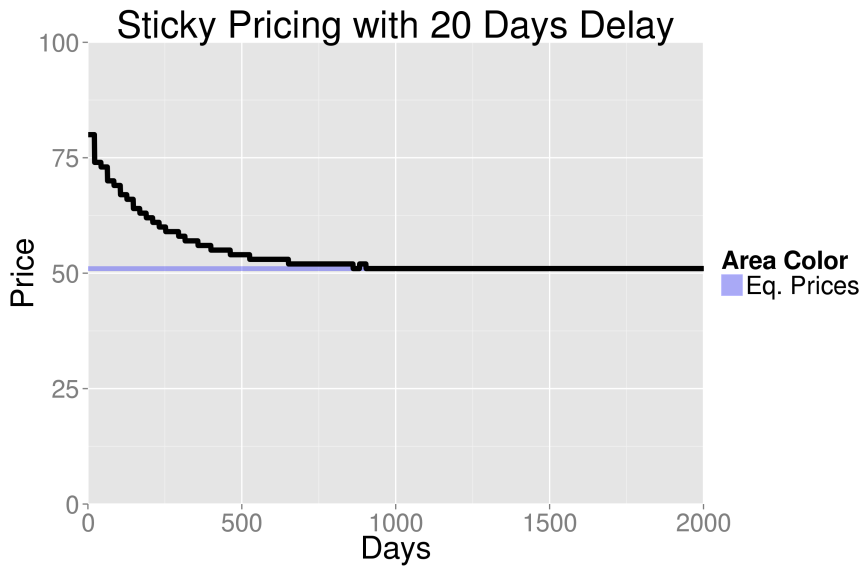

Figure 4: Like in figure 3 $\delta = 20$ but this time the trader adjusts her prices only every 20 days ($s = 20$). Notice that the time between one price change and the next is irregular, this is because there is a fixed $\frac{1}{s}$ chance of activating the PI controller each day. The result is the same approach as figure 1 but in a longer time frame.

I defined price stickiness s as the average number of days the agent waits before activating its PI controller to change prices. Define timidity z as the number dividing the baseline PI parameters (so a timidity of 10 with a baseline of 0.1 means that the PI parameters $a = b = \frac{.1}{10} = .01$). The PI formula in equation 2 changes to:

$$p_{t+1} = \begin{cases} p_t, & \text{with probability } 1 - \frac{1}{s} \\ \frac{a}{z} e_t + \frac{b}{z} \sum_{i=1}^{t} e_i + p_0, & \text{with probability } \frac{1}{s} \end{cases} \tag{3}$$

The larger the z (with $z > 1$) the more cautious the controller becomes. This is because the errors now get multiplied by a smaller number and therefore have a smaller effect on prices.

Fix the demand delay δ to 50 (this is in order to better show the interplay between stickiness and timidity). Figure 6 shows which combinations of timidity and stickiness achieve correct prices over 5 experimental runs. Define the daily distance from the correct price as:

$$\sum_{t=1}^{T} (p_t - p^e)^2$$

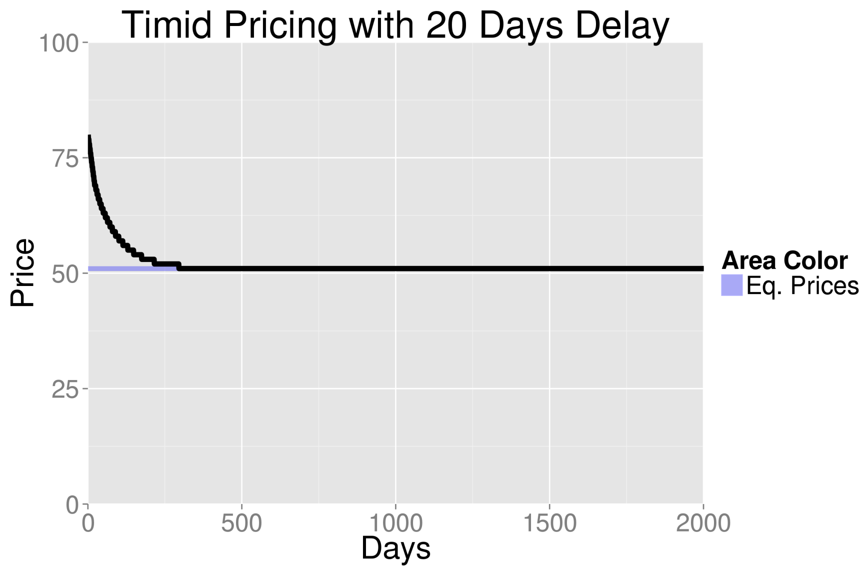

Figure 5: Like in figure 3 $\delta = 20$ but here PI controller has $a = b = .01$, 10 times smaller than the original (that is $z = 10$).

Using the demand $q_t = 101 - p_{t-50}$ the equilibrium price is $p^e = 51$. Figure 7 shows for which combination the distance is minimized. That is which combination of timidity and stickiness achieves the fastest convergence to correct prices.

In this paper I will focus almost exclusively on stickiness s rather than timidity z. Moreover s will be fixed and exogenous for most examples. However I will show in section 8 how the endogenize s and let the firm set it on its own.

Agents working by trial and error benefit from acting slowly and timidly whenever price changes take time to have an effect. Since there are no menu costs, the agents are indifferent between adjusting the price often but timidly or seldom but aggressively. The weakness of this section is that the delay is arbitrarily fixed and exogenous. The delay will become endogenous as supply chains are introduced.

3.4 We can reduce knowledge further with a minimum inventory buffer

One weakness of the error we feed to the PID controller is that equation 1 assumes netflow can go negative. If the seller manages to sell all its stock, she must estimate how many more goods she would have been able to sell at that price. This is unrealistic. An alternative is to hold a minimum inventory, as

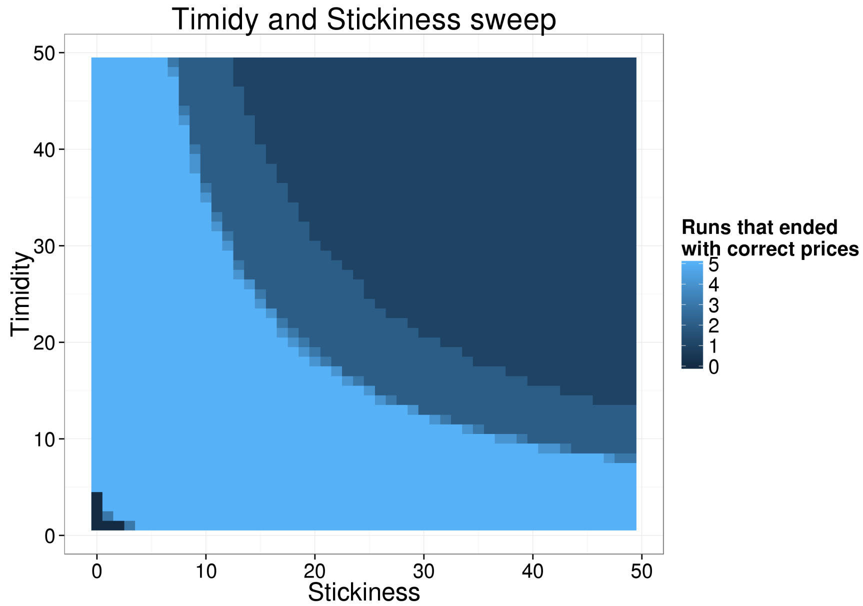

Figure 6: Run the model 5 times for 15000 market days with fixed PI parameters and speed but different initial prices. Controllers that are too fast (no stickiness or timidity) or too slow (too much stickiness or timidity) fail in at least some cases. Demand delay is 50 days

this allows the firm to sell more than what is produced in a day and so learn whether the price is too low.

The advantage of having an inventory is that we can change the error we feed in the controller from

$$e_t = \text{Outflow} - \text{Inflow}$$

to just

$$e_t = -\Delta\text{Inventory}$$

The disadvantage is that the seller must build up an inventory.

Remember that in this section the seller receives a fixed amount of goods q^* every day. She then needs a strategy to stock up a sufficient level of inventory. Define i^* as the minimum buffer inventory level. The PI controller targets 0 sales as long as actual inventory is below i^* and targets $\Delta\text{Inventory} = 0$ otherwise. In other words inventory accumulation takes precedence. Excluding the initial days of stocking up the dynamics of this controller are exactly the same as those shown above. Traders will use inventory buffers for the rest of the paper.

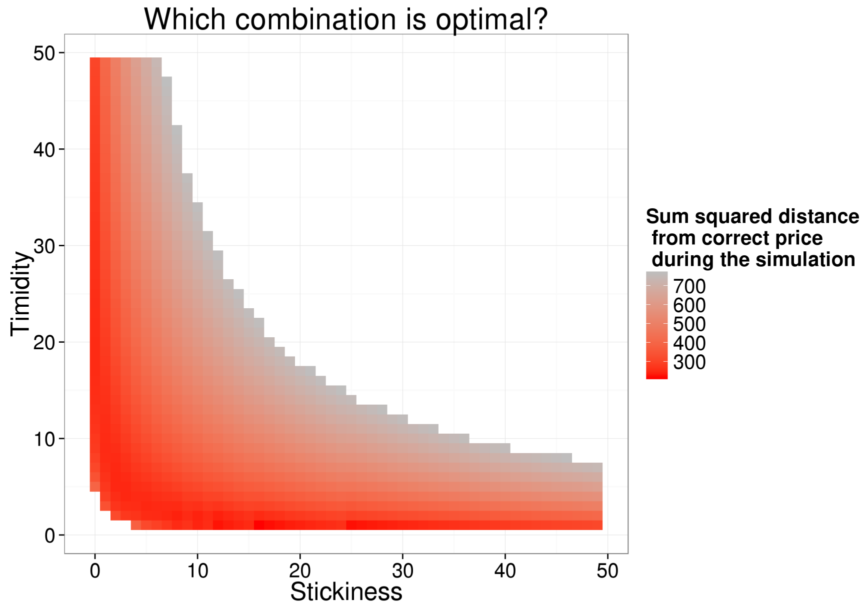

Figure 7: Average sum squared distance over 5 simulations run. The minimum distance is achieved by stickiness of 16 and timidity of 1. Only the successful combinations from figure 6 are considered. Demand delay is 50 days

3.5 Smoothing prices through moving averages does not solve demand delays

I showed how demand delays cause prices to oscillate as in figure 3 and then how stickiness and timidity can overcome such issues. Because the price oscillation is very regular in the example shown, it might seem possible to avoid dealing with stickiness and instead smooth the PI controller's prices through a moving average. This unfortunately does not work: moving averages do not solve oscillations caused by demand delays.

Notice first that there are two variables we can smooth. Either smooth the error e_t to feed into the PI controller or the policy p_t that comes out of it. In control theory, these are called respectively 'process variable filtering' and 'controller output filtering'. Filtering e_t through an arithmetic moving average makes the delay problem worse. This is because averaging out e_t with previous values make the PI controller deal with a delayed version of its input. The demand delay is what fooled the PI controller into oscillating its prices in the first place and smoothing e_t increases the delay faced by the PI controller and therefore the amplitude of its price oscillations.

Understanding why smoothing the PI output p_t also has no positive effects help explain why stickiness and timidity work instead. Demand delays mean that the error e_t fed into the controller does not accurately reflect the effect of the price p_t on the demand. Over time the errors e_t accumulate in the integrative part of the controller, that is $b\sum_{i=0}^{t} e_t$. It is the progressive increasing and then unwinding of the integrative part that causes prices to oscillate. Stickiness fixes this by only updating the integral part approximately every s days, timidity fixes this by dividing the effect of the integrative part of the controller. Smoothing out the PI output instead has no effect on the integrative controller; it slows down the output of the controller but not the accumulation of errors in its integrative part. The PI controller has a sluggish output but all this does is to give more time to the integrative part of the controller to increase and eventually this amplifies the price oscillations.

Figure 8 shows the prices generated by a PI controller filtering either its input or output by a 20-day moving average when facing a demand delayed by $\delta = 20$ days. The prices generated are not better and the oscillations are deeper, regardless of the filter used. Stickiness and timidity work better than filtering.

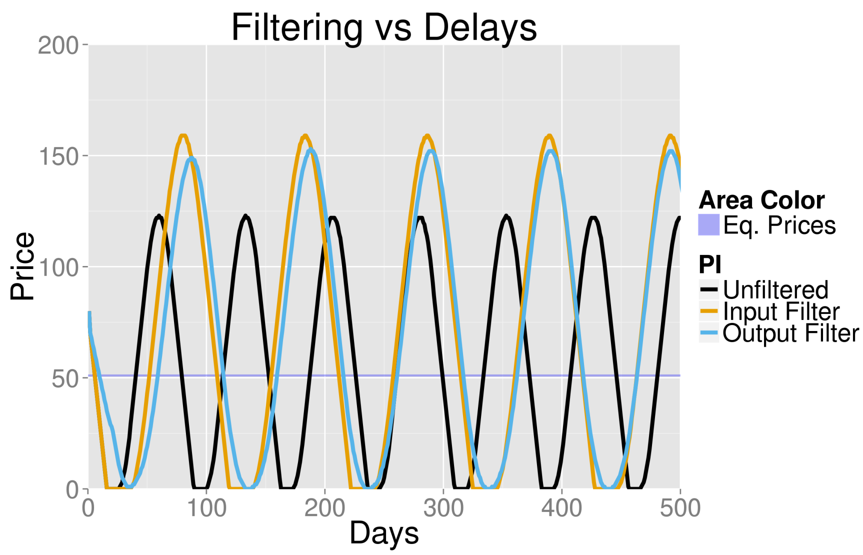

Figure 8: I compare here the effect of adding a 20-days moving average filter to the input e_t or the output p_t of a PI controller facing a 20 days delayed demand. The filters do not improve the controller output and in fact increase the amplitude of the price oscillations

4 Firms and Production

In this section I expand the Zero-Knowledge methodology in two directions. First I replace the previous section's exogenous fixed daily inflow with endogenous production targets. Second I allow the agent to act in multiple markets at the same time, hiring workers while selling output. Assuming the agent knows whether it is in a monopolist or competitive market the agents reach the correct production levels and prices.

4.1 Independent PI controls coupled with simple marginal analysis can simulate one-sector competitive and monopolist markets

Agents in this section produce their own goods for sale. These agents I call the Zero-Knowledge Firms. In parallel they have to hire workers, buy inputs, and price their output. They are price-makers when selling or setting wages and price-takers when buying other inputs. Each price is set by an independent PI controller as in section 3.1.

Production is linear with respect to workers hired L_t:

$$F(L_t) = L_t$$

The firm has to decide how many workers to hire. The simplest way to do so is to raise production as long as:

$$\text{Marginal Benefit} > \text{Marginal Cost}$$

More precisely: a firm producing one type of good priced p_t and consuming labor as only input with unit wage w_t maximizes the following profit function:

$$\Pi_t = p_t q_t - w_t q_t$$

Where p_t and w_t are themselves function of production q_t so that maximum profits are achieved when:

$$p + q\frac{\partial p}{\partial q} = w + q\frac{\partial w}{\partial q}$$

Now define $\mu_p = \frac{\partial p}{\partial q}$ the price impact of increasing production, that is by how much sale price goes down when production goes up by one unit. Similarly define $\mu_w = \frac{\partial w}{\partial q}$ as the wage impact of increasing production, that is by how much wages need to increase in order to hire enough workers to produce one more unit of good. So that at any point in time we want to set the production target such that:

$$p_t + \mu^p L_t = w_t + \mu^w L_t \tag{4}$$

This is a decision rule for production targets that is based on daily prices p_t and w_t which the PI controllers discover.

As shown in section 3.1 it takes some time for PI controllers to find the correct prices. Because marginal benefits and costs are computed with PI prices, production decisions should be taken infrequently to give the controllers time to be correct. In particular as long as $p_t + \mu^p L_t > w_t + \mu^w L_t$ increase production targets by 1 unit of output or lower it by 1 unit of output when $p_t + \mu^p L_t < w_t + \mu^w L_t$

Define T as the decision period: how often, in days, the firm checks whether to change production. It is set to 20 for all simulations. Much like with stickiness s, T is also stochastic: there is a fixed chance of $\frac{1}{T}$ each day of choosing a new production target.

The firm must also know what the price impacts μ are. I will deal with their endogenous discovery in section 7. Until then I'll simply assume they are known. To a competitive firm, price impacts are always 0. To a monopolist, price impacts equal the demand and supply slopes.

Take a firm facing the daily demand function: $q = 102 - p$, with daily production function[4] $q = f(L) = L$ and wage curve $w = 14 + L$. A firm acting as a monopolist would maximize profits by producing 22 units a day and selling them at \$80.

Figure 9 shows a sample run of a Zero-Knowledge monopolist firm. Notice first that the monopolist starts producing after 250 days. This is the time it takes for the PI controller setting wages to find the right wage for 1 worker (\$15). Notice also that there is no noise once reaching equilibrium.

In perfect competition, equilibrium is a daily total production of 44 units sold at \$58. Figure 10 shows a sample run with 5 competitors. Multiple agents create noisy results due to coordination failure. While each firm might see an increase in production as profitable, the demand is not enough when all firms increase production at the same time.

Notice that the number of firms competing is not important for the correct result. If I run the model with a single firm with zero price impacts I will get a noiseless perfect competitive solution. Perfect competition here is purely a function of the internal production decision processes of the firm. In section 7 though the firms will learn on their own their price impacts and having competitors will force them to zero their price impacts.

4.2 Competitive markets micro-structure is more confusing than its aggregate equilibrium suggests

Competitive scenarios with multiple agents, as in figure 10, achieve quasi-equilibrium: prices and quantity hover around the equilibrium levels. However the firms that compose this aggregate equilibrium are subject to more chaotic dynamics. Each day there is usually a single price-leading firm that sells to all the customers while the other firms sell nothing. This monopoly lasts only for a day and a different price-leading firm caters to all the customers the day after. This is in spite of the fact that each firm targets only a fraction of the total production.

[4] The demand and supply parameters here and in the following sections are chosen exclusively so that the equilibrium is in natural numbers

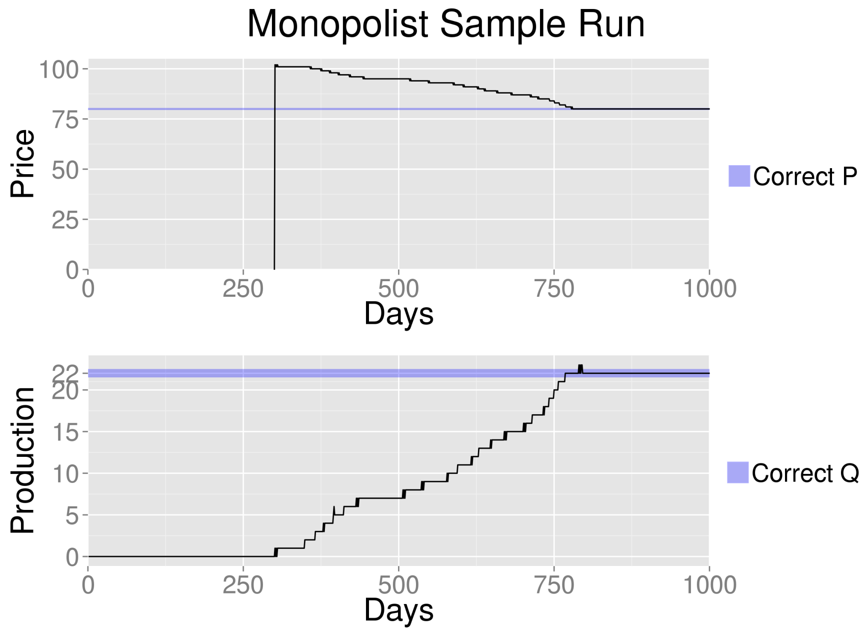

Figure 9: A sample run with a monopolist firm. It reaches the correct price and quantity

More specifically each firm uses its PI controller to set their price each day. One of the firms will by necessity price their goods slightly below the others. Because of perfect competition that firm will attract all the customers available. The firm will sell more than its targets burning through its buffer inventory. The day after the firm that sold too much will increase its price (as dictated by its PI controller) while all the other firms will lower theirs since they didn't hit their target. A new firm then takes the pricing lead and the cycle starts over.

At its core, this is an information asymmetry problem. Firms know nothing of other firms while customers know everything about them. So that when a firm drops its price the customers react immediately while competitors can only do so after their consumer base has vanished. It is a dynamic that is enabled by the existence of inventories firms can dip into. With no inventories the price-leading firms would only be able to supply a limited quantity of goods and multiple daily prices would emerge.

More generally perfect competition is the hardest market structure to model through trial and error pricing. The more market power a firm has, the more informative trial and error pricing is. Small changes in prices for a monopolist generates small changes in quantity demanded, but small changes in prices in a perfect competitive market sometimes generate large swings in demand and sometimes no change at all. But I believe it is important to show

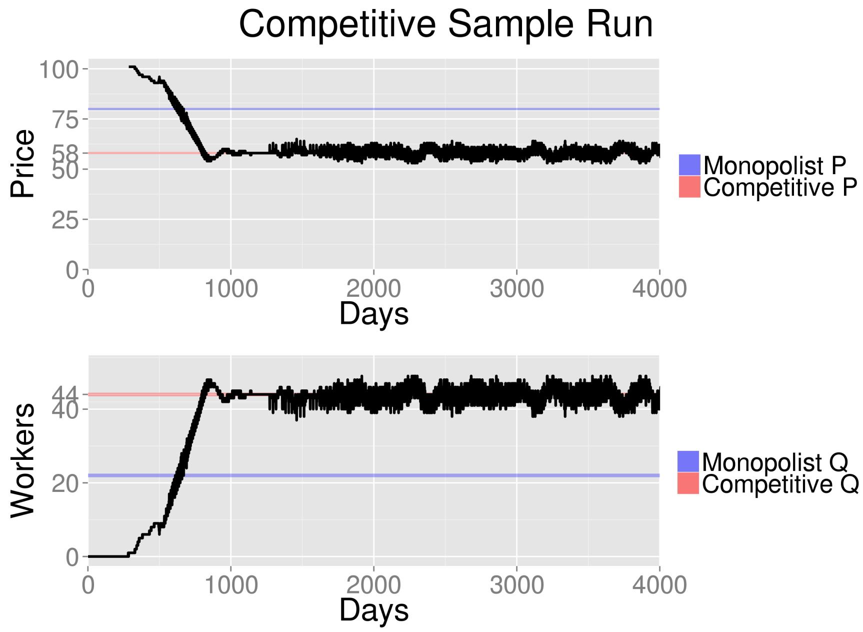

Figure 10: A sample run with a 5 competitive firms. There is noise but centered around the correct price and quantity.

that even in the worst case scenario for trial and error Zero-Knowledge traders still get to the equilibrium successfully albeit only in an aggregate sense.

For the rest of the paper I will continue using multiple agents within the same competitive market in spite of the noise and the fact that they could be replaced by a single agent forced to act competitively, that is with price impacts $\mu = 0$. This is because when I allow agents to learn price impacts in section 7 there need to be more than one for them to act competitively. Adding multiple agents only from that section on would generate two kinds of noises at once: competition and learning and that would make comparison between learning and non-learning agents impossible.

5 Supply Chains

I place the Zero-Knowledge firms of the previous section in a supply chain. Because it takes time for firms downstream to change production targets, firms upstream face a delayed demand similar to section 3. Much like that section, the trial and error pricing creates oscillations and fails to reach equilibrium. Much like that section, price-stickiness solves such issues.

5.1 Zero-knowledge firms in a supply chain create endogenous delays that break the model

Take a supply chain made of two sectors: wood and furniture. There is a final daily demand for furniture which is exogenous and fixed at:

$$q_F = 102 - p_F$$

Daily production of one unit of furniture requires one worker and consumes one unit of wood:

$$q_F = \min(L_F, q_W)$$

Daily production of one unit of wood requires one worker.

$$q_W = L_W$$

Each sector has its own independent linear labor supply:

$$w_W = L_W$$
$$w_F = L_F$$

Helpfully, there are infinite trees waiting to be cut.

In this section further assume that the wood sector is monopolized while the furniture market is competitive. I go through all the market-structure permutations in section 6. Solving for the market equilibrium yields the following:

$$q_F = q_W = 17 \tag{5a}$$
$$w_W = w_F = 17 \tag{5b}$$
$$p_W = 68 \tag{5c}$$
$$p_F = 85 \tag{5d}$$

The theoretical demand for wood from the furniture sector is:

$$p_W = 102 - 2q_W \tag{6}$$

What I will do next is the following: I will find the best PI parameters that would deal the demand in equation 6 if it were a single independent market. I will then show how such parameters do not reach equilibrium in a supply-chain. Finally I will show that adding timidity and sticky prices solve the oscillations and achieve the market equilibrium.

Figure 11 shows the parameter sweep for the optimal PI controller of a monopolist facing the undelayed demand function 6; for each parameter I show the average simulated log_{10} sum squared errors. The parameters with the lowest error are $a = 0$ and $b = 2$. This makes sense since 2 is the slope of the demand curve.

I have shown that $a = 0, b = 2$ are the optimal PI parameters for the wood producer when facing the fixed demand in equation 6. Now I let the same PI controller face the same wood demand but this time it is generated

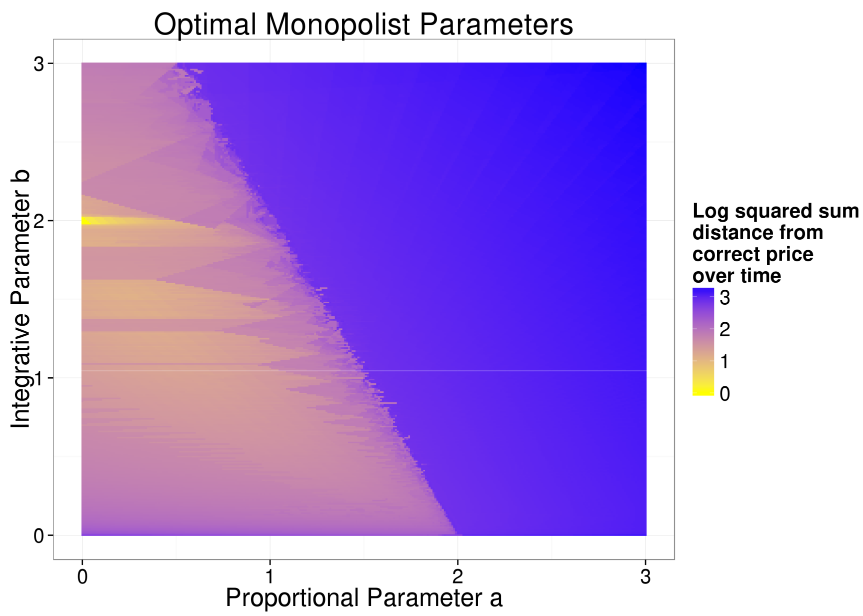

Figure 11: The average squared distance from correct prices when a monopolist faces demand $p = 102 - 2q$ and labor supply $w = L$. Each cell represents a pair of parameters used by the monopolist's sales PI control. The optimal parameter pair is predictably $a = 0, b = 2$ reflecting the underlying demand.

by a downstream market sectors made up of other Zero-Knowledge firms. The effects are shown in figure 12. Far from being optimal, neither production nor prices ever reach equilibrium and prices -especially for wood- oscillate.

The parameters that were optimal when facing an exogenous demand prove too aggressive when the same demand is made up of other Zero-Knowledge firms. The issue is delay. It takes time for furniture producers to notice new wood prices and change their production. By the time furniture consumers adjust, the producer has changed the sale price again. These delays downstream feed into the upstream trial and error loop. The wood price swings from 0 to over 100 because the wood monopolist's netflow reacts slowly to changes in prices.

Table 1 shows the decisions made by the PI controller setting the prices of the wood producer from day 1437 to 1445 and shows what causes the price swings. The root cause is the difference between what is produced and what is consumed. The wood producer is producing 15 units of wood a day but only sells 8 or 9. In fact the wood producer is targeting an even higher production of 16 because the prices are still very high. The PI controller has to find a price such that it can sell all 15 units of good; as a matter of fact that is impossible to do so in a short amount of time because the production decisions

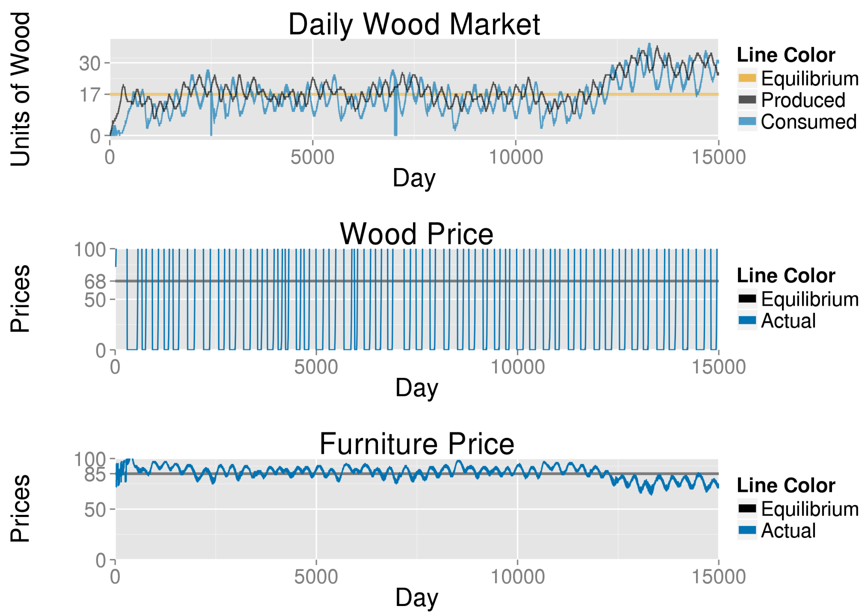

Figure 12: A sample run of the supply chain model using the PI parameters that were optimal when the demand was immediately reacting.

downstream respond to prices too slowly . Notice that the PI controller here is just $p_{t+1} = 2\sum_{i=0}^{t} e_i$ so the value of the first column (p_t) is just the twice the value on the last column ($\sum e_i$).

Table 1: The price and production decision of the wood producer in figure 12 from days 1437 to 1445

Day	p_t	Target Production	Production	Sales	e_t	$\sum e_i$
1437	94	16	15	10	-5	47
1438	82	16	15	9	-6	41
1439	70	16	15	9	-6	35
1440	58	16	15	9	-6	29
1441	46	16	15	9	-6	23
1442	34	16	15	9	-6	17
1443	22	16	16	9	-7	10
1444	6	16	16	8	-8	2
1445	0	16	16	8	8	-6

The PI controller is aggressively cutting prices but these have very little effects in the short run. Eventually low prices do increase demand downstream and decrease production upstream but by then the prices are at a level too low (0$ in fact) and demand outstrips supply (with inventories being bought instead). This mismatch between the speed of production adjustment and price adjustment is why equilibrium is not achieved. It is clear that prices need to adapt more slowly.

Figure 13 shows a second example, this time with timidity $z = 10$. This is the same as lowering the b parameter from 2 to 0.2. The equilibrium is not achieved and the oscillations are still present. I will now show how adding price stickiness improves the dynamics.

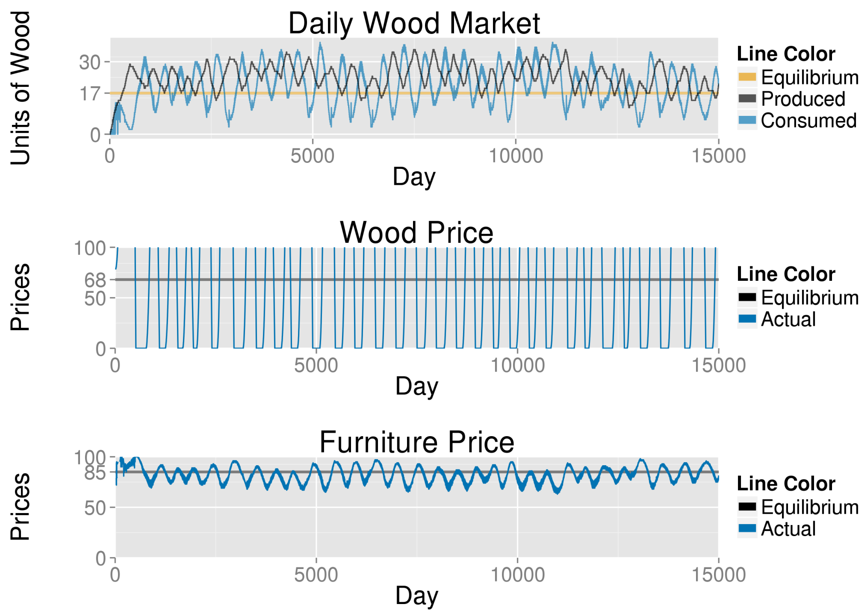

Figure 13: A sample run of the supply chain model using the PI parameters that are 10 times more timid than the optimal.

5.2 Adding price-stickiness to the upstream firm restores equilibrium

Take the previous PI controller add both a timidity $z = 10$ and a price stickiness of $s = 50$ days. As figure 14 shows, these parameters are enough to fix the supply chain: production and prices in both sectors are the correct ones and remain in equilibrium. This is because now prices change slowly enough for production to fully adapt to them.

Sticky prices eliminate bullwhip effects the same way they dealt with arbitrary demand delays in section 3.3. The difference is that in section 3.3 the delays were exogenous as I added them just to show how to deal with them. In this section instead there is a demand delay but it is caused by the interaction between upstream and downstream firms.

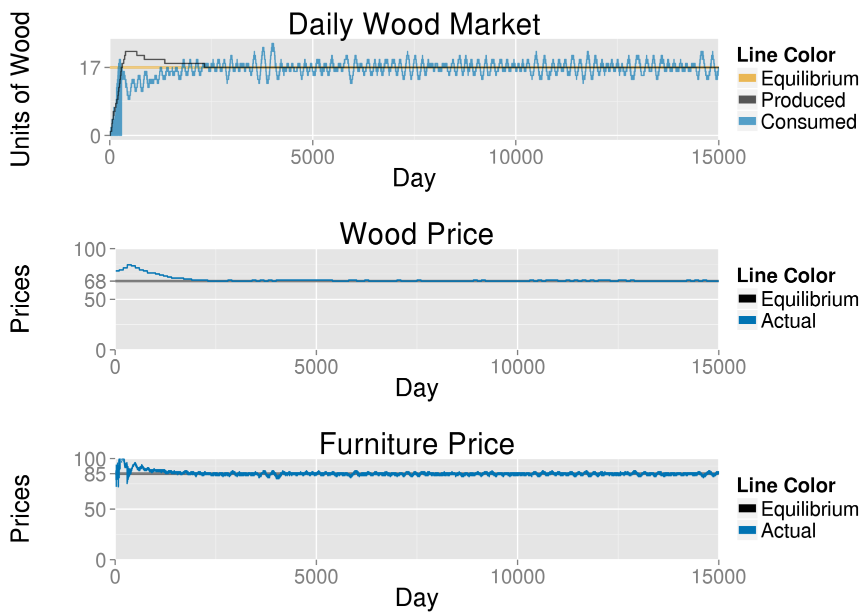

Figure 14: A sample run of the supply chain model where the wood producer uses sticky prices.

There are two endogenous sources of delays in the supply chain. The first delay is the time it takes between a firm making the decision to change its production quota and the controllers adapting to it by finding new wages and prices. The second delay source is the decision period T of the furniture producers (how quickly they change production targets given current prices) which delays their response to change in prices of the wood supplier. The larger T downstream the higher the upstream price stickiness s need to be to balance. Figure 15 shows the relationship between the stickiness s and T.

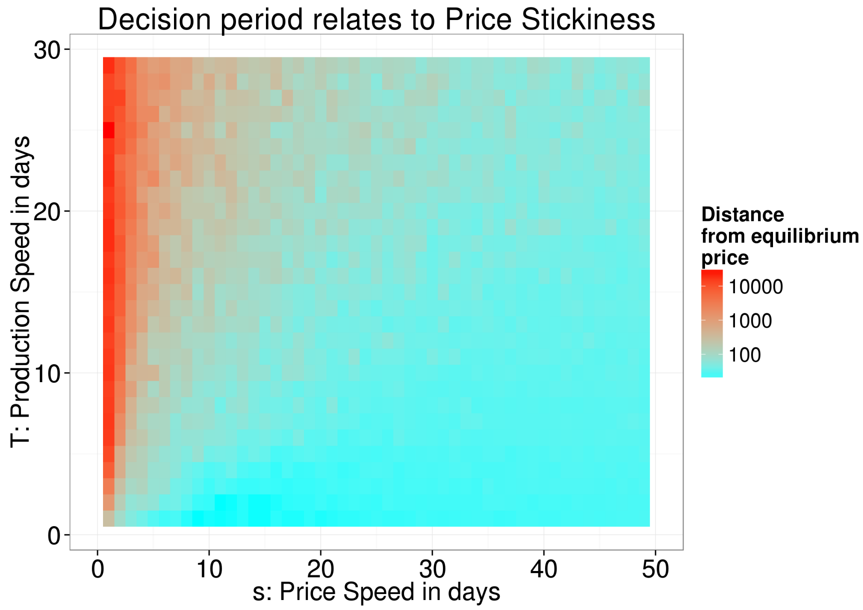

Figure 15: The decision period T(in days) of firms is an important source of delays in the system; the higher T the higher the price stickiness needs to be in order to balance it. Each tile represents a 5-runs average squared distance from the correct price over the whole run.

6 Market Structure

In this section I go through the market structure permutations to show how the Zero-Knowledge firms can adapt to it and reach equilibrium. All these simulations use the same price stickiness as in the previous section.

In equation 5 I expressed the solution where the wood sector is a monopolist while the furniture sector is competitive. If the wood sector is competitive while the furniture is monopolistic the equilibrium is:

$$q_F = q_W = 17 \tag{7a}$$
$$w_W = w_F = 17 \tag{7b}$$
$$p_W = 17 \tag{7c}$$
$$p_F = 85 \tag{7d}$$

If both sectors are competitive the no-profit equilibrium should be:

$$q_F = q_W = 34 \tag{8a}$$
$$w_W = w_F = 34 \tag{8b}$$
$$p_W = 34 \tag{8c}$$
$$p_F = 68 \tag{8d}$$

I run 100 simulations for each market structure (competitive means 5 firms in the same sector). Each simulation runs for 15000 market days. Firm have an inventory buffer of 100, regardless of market structure. All input producers use sticky prices (50 days each price change), regardless of market structure.

In general the model behaves as predicted by theory. Figure 16 shows the distribution of input prices at the end of the simulation; figure 17 shows the output prices; figure 18 shows the quantity produced.

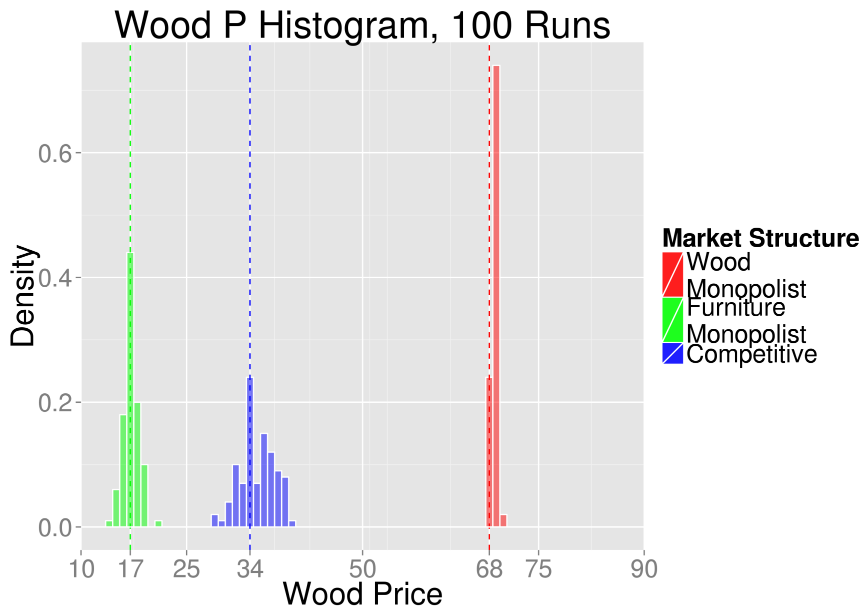

Figure 16: The price of wood (first sector) for 300 simulated runs, 100 for each market structure. The vertical dashed lines represent the theoretical equilibrium. Each datum in the histogram is the price on the last day of the simulation.

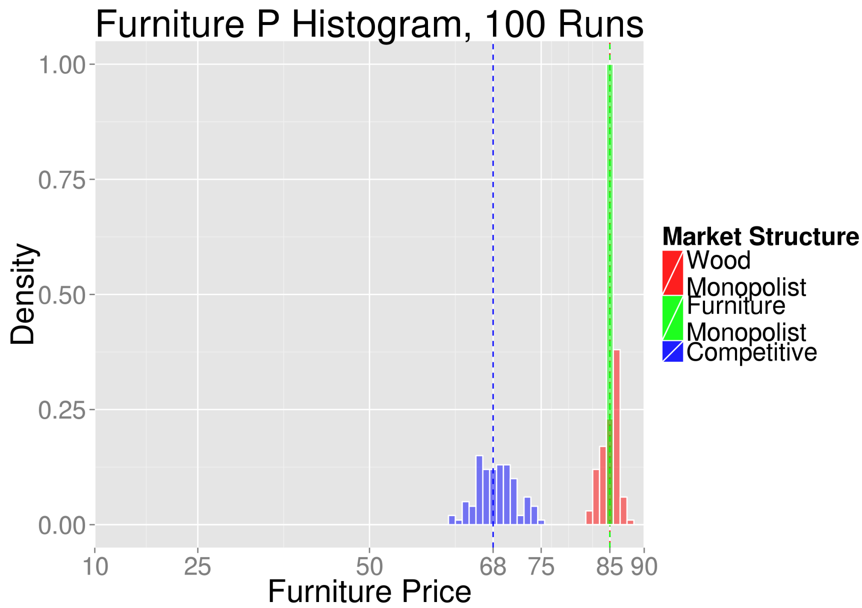

Figure 17: The price of furniture (second sector) for 300 simulated runs, 100 for each market structure. The vertical dashed lines represent the theoretical equilibrium. Each datum in the histogram is the price on the last day of the simulation.

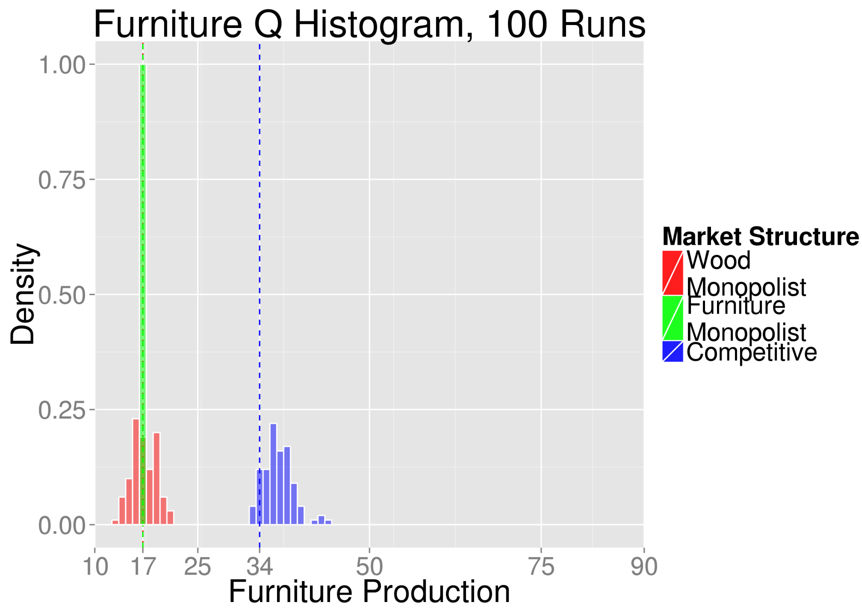

Figure 18: The units of furniture produced daily at the end of 300 simulated runs, 100 for each market structure. The vertical dashed lines represent the theoretical equilibrium. Each datum in the histogram is the production on the last day of simulation.

7 Learning Price Impacts

The results from section 4 depends on the firms knowing whether they are in a monopolist or competitive market. In this section I remove this assumption by allowing firms to learn on their own the price impacts they face. Learning produces noise compared to the results in the previous section, but the equilibrium quantities and prices are comparable. The only exogenous constraint remaining in this section is that upstream firms use sticky prices.

7.1 Regressing workers on price works well in a one-sector economy

A Zero-Knowledge firm should be able to learn its own market power. In previous sections the price impacts were given, now firms discover them.

The firm takes the price generated by its PI controls and regresses it against number of workers. The regression identifies how much increasing production changes prices. Zero-Knowledge firms use two regressions side by side. First, they fit one-step error correction regression model (Banerjee, Dolado, Galbraith, & Hendry, 1993):

$$\Delta p_t = \beta_0 + \beta_1 \Delta L_t + \beta_2 p_{t-1} + \beta_3 L_{t-1} + \epsilon \tag{9}$$

Where p is price and L are workers hired. The firm identifies the long run relationship between the two variables and use it as approximate price impact:

$$\mu^p = -\frac{\beta_3}{\beta_2}$$

The second regression is the linear model

$$p_t = \gamma_0 + \gamma_1 L + \epsilon \tag{10}$$

Where the price impact discovered is:

$$\mu^p = \gamma_1$$

Each day the Zero-Knowledge firm selects the regression that better predicts today's price. If a firm trades in multiple markets (for example selling furniture, buying wood and hiring workers) then it has multiple regression pairs, each focusing on predicting one price (output price, input price, wages). For the input markets where the firm is a price-taker the paid price is used in lieu of the PI one.

Because the PI controls generate one observation each day in each market it makes sense to implement the regressions by a Recursive Least Squares filter(Welch & Bishop, 1995). Take as example Equation 9. It has four parameters: $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3)$. Each day the firm observes the price offered, the labor hired and their lags $y_t = \Delta p_t$,$\mathbf{x}_t = (1, \Delta L_t, p_{t-1}, L_{t-1})$. The current estimation of $\boldsymbol{\beta}$ is $\hat{\boldsymbol{\beta}}_{t-1} = \left(\hat{\beta}_{0t-1}, \hat{\beta}_{1t-1}, \hat{\beta}_{2t-1}, \hat{\beta}_{3t-1}\right)$. Update it with the new observation in four steps:

$$\mathbf{k} = \mathbf{P}_{t-1}\mathbf{x}^T\left(\mathbf{x}P_{t-1}\mathbf{x}^T + 1\right)^{-1} \quad \text{Constructing the Kalman gain} \tag{11a}$$
$$\epsilon_t = y_t - x\hat{\boldsymbol{\beta}}_{t-1} \quad \text{Finding the prediction error} \tag{11b}$$
$$\hat{\boldsymbol{\beta}}_t = \hat{\boldsymbol{\beta}}_{t-1} + \mathbf{k}\epsilon_t \quad \text{Updating predictor given error} \tag{11c}$$
$$\mathbf{P}_t = (I - \mathbf{k}\mathbf{x}_t)\mathbf{P}_{t-1} \quad \text{Updating covariance matrix} \tag{11d}$$

Where $\mathbf{P}_t$ is the 4×4 covariance matrix. Functionally $\mathbf{P}_0$ is a Bayesian prior which I set at $10^4 I$ for all simulations.

Figure 19 and figure 20 show the results of running 100 simulations with a learning monopolist and 100 simulations with 5 learning competitors. Firms learn their market power correctly, which involves learning whether they are a monopolist or not and if they are what the slope of the demand actually is.

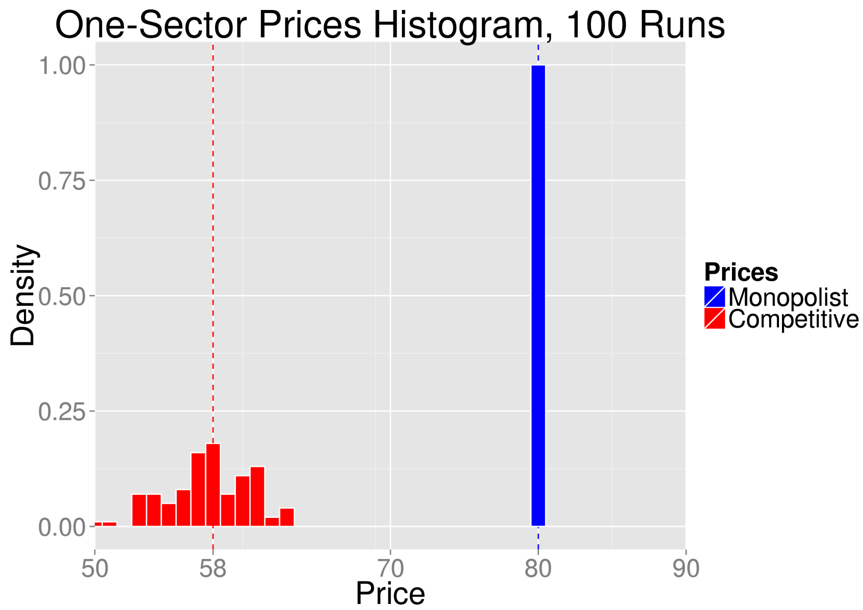

Figure 19: The histogram of prices from running 100 monopolist and 100 competitive (5 firms) scenarios. All firms need to learn the price and wages impact. All firms target inventory (100 units of output). Each observation in the histogram is the price on the last day of the simulation

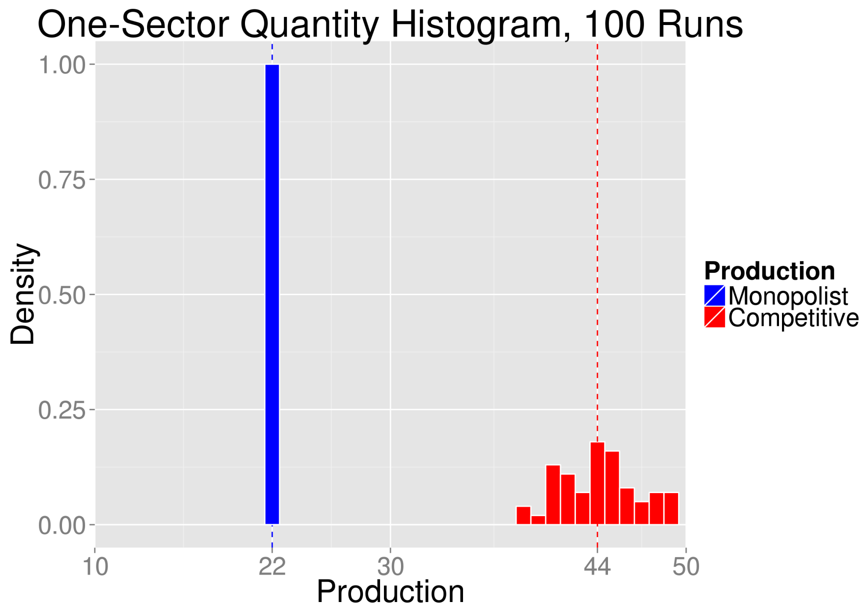

Figure 20: The histogram of simulated production obtained by running 100 monopolist and 100 competitive (5 firms) scenarios. All firms need to learn the price and wages impact. All firms have a buffer inventory (100 units of output). Each observation in the histogram is the production on the last day of simulation

7.2 Learning in a supply-chain is harder and less effective

Learning is far more problematic in a supply-chain. First, if there is a delay δ between setting a price p_t and it affecting quantity traded, the Zero-Knowledge firm should regress $p_{t-\delta}$ over L_t. But this is impossible as the delay is unknown. Secondly, because of stickiness, the firm often sets prices p_t that are not the market clearing ones. Because learning works by regressing paired p_t and L_t, the results are often useless. Figures 21, 22, 23 show the results of 100 simulations for each market structure. All are using buffer inventory and sticky prices. The results are far more dispersed, usually because one of the recursive least squares filters failed to learn the correct slope.

7.3 Circular causality and passivity are the main learning weaknesses

The error-correcting model assumes that labor determines prices. That is true as the PI control reacts to increased production by lowering prices. But production also responds to prices as firms fire workers when sale prices fall.

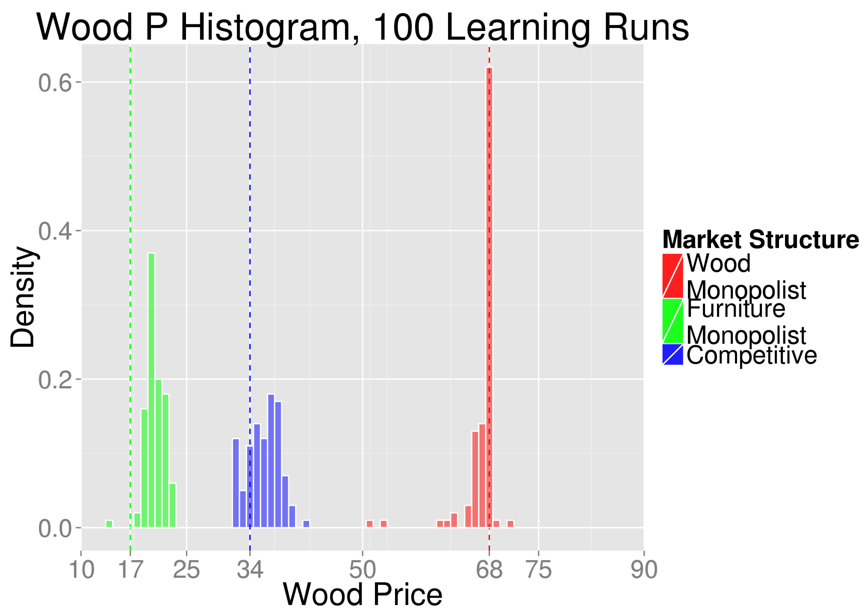

Figure 21: The price of wood (first sector) for 300 simulated runs, 100 for each market structure. The dashed vertical lines represent the theoretical equilibrium. Each datum in the histogram is the price on the last day of the simulation

Circular causality is exacerbated by the regression itself since the slope found is part of the profit maximization function linking the two variables.

The root cause of this confusion is that agents are passive learners when it comes to price impacts. Zero-Knowledge firms observe long time series of prices and production and try to make sense of them. What these firms never do is willfully experiment with wrong prices. Firms never try to double prices to see the effect on demand or increase production beyond the optimal level to test their estimated labor supply slope. Agents are, in meta-heuristics term, greedy. They always exploit current knowledge and never explore. This I believe is in line with how learning is usually modeled in economics (Evans & Honkapohja, 2009) but it is probably an assumption that should be dropped in future papers.

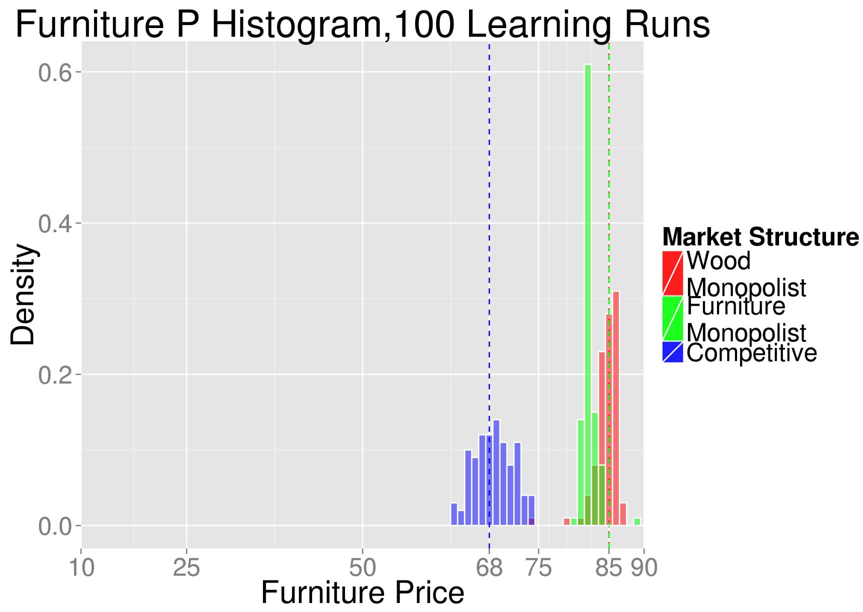

Figure 22: The price of furniture (second sector) for 300 simulated runs, 100 for each market structure. The dashed vertical lines represent the theoretical equilibrium. Each datum in the histogram is the price on the last day of the simulation.

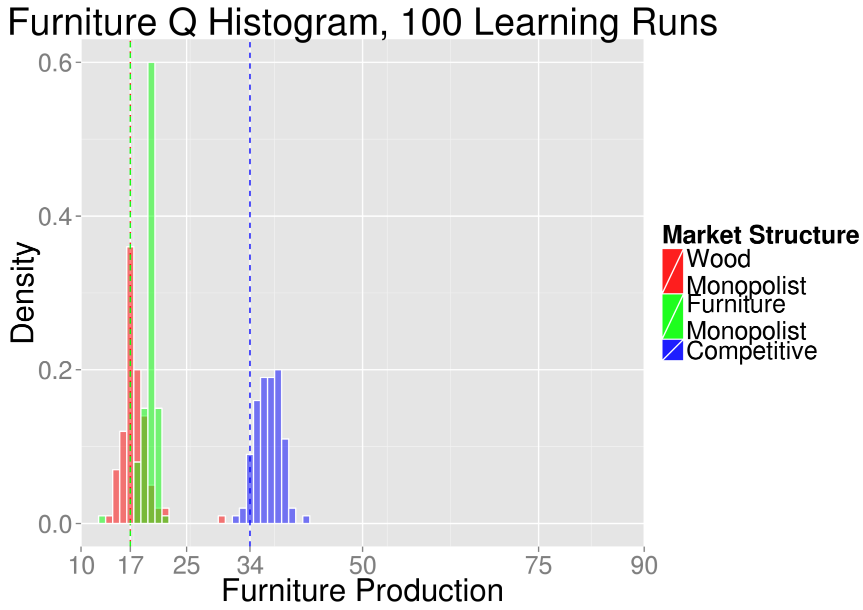

Figure 23: The units of furniture produced daily at the end of 300 simulated runs, 100 for each market structure. The dashed vertical lines represent the theoretical equilibrium. Each datum in the histogram is the production on the last day of the simulation.

8 Learning stickiness

In this section I remove a further assumption on firms behavior. I introduced stickiness s in section 3.3 but I always set it exogenously: firms would either have stickiness or not. In this section s emerges endogenously by providing the firm with a way of setting it on its own.

The firm needs to change the stickiness parameter of its PI controller while it is in use. This is the domain of adaptive control (Landau, Lozano, M'Saad, & Karimi, 2011). In this case too Zero-Knowledge firms act by trial and error.

The first step involves defining a performance metric to judge controllers and their parameters. Here I use the integral time absolute error (ITAE) performance index (Shinners, 1998):

$$\sum_{i=1}^{M} i\,|\hat{e}_{t-M+i}| \tag{12}$$

The lower the ITAE the more precise the controller. M is the time horizon and the error e_t is the PI error as defined in equation 1. The performance index simply states that PIs are better parameterized if the system is on target and being on target in the long run matters more than in the short run.

In this paper I only focus on the stickiness parameter: how many days pass between each adjustment by the PI controller. I modify this parameter by simple hill-climbing (Luke, 2009). Zero-Knowledge firms set a stickiness and test it for $M = 100$ days. If it has better performance than the previous stickiness then we keep it otherwise we revert back to the previous one. This process loops forever.

A sample run where the firm tunes its stickiness is in figure 24. The firm starts tuning its stickiness after 1000 days, it changes stickiness in steps of 5. In this run there is a first abortive attempt at sticky prices at around day 2000. The experiments fail because prices get sticky while out of equilibrium which cause poor performance. After reaching equilibrium stickiness stops mattering which results in the stickiness parameter bouncing between 20 and 25 days.

The tuning process could be improved by making learning forward-looking. This is generally called indirect adaptive control. The idea is to fit process data to a statistical model and then estimate performance by treating the fitted model as the real one. The issue is usually to find the right statistical model. See paper 12 of Landau et al. (2011) for a primer on the field. Hill-climbing bypasses that by experimenting directly over the real system.

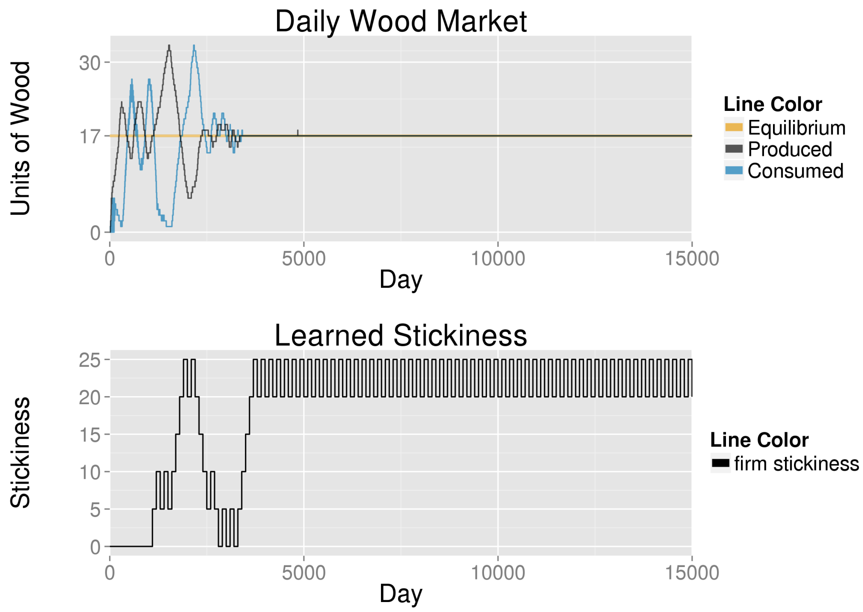

Figure 24: A sample run where the firm starts selling with a non-sticky PI controller $b = 0.2$. We start tuning after 1000 days. Time horizon M is 100 days.

9 Fitting Zero-Knowledge Traders to Data

In this section I show how to fit PI controllers to datasets and how to use my model empirically. The data quality required is unfortunately high.

While PI controllers aren't used directly to price goods in the economy we might want to estimate what are the best PI parameters that simulate empirical pricing. We need two time series, the price set by the controller and the error the controller reacted to. The standard technique to fit a PI controller to data is to turn it into a velocity form (Åström & Hägglund, 1995) where the PI formula becomes:

$$\Delta u_t = \alpha e_t + \beta e_{t-1} + \gamma e_{t-2}$$

Which can then be fitted through ordinary least squares (OLS).

I am not pursuing that strategy here because it is brittle: OLS fails once I add windup stop or any other modification to the general PI controller. I fit the PI parameters by simulation instead. I start with a random vector of PI parameters a, b and then generate the simulated policy time series $\tilde{p}_t$ given the error time series e_t. Finally I compare the simulated time series with the real policy time series p_t and record the absolute simulated error:

$$\epsilon = \sum_{i=1}^{T} |\tilde{p_i} - p_i|$$

This gives us a mapping $(a, b) \to \epsilon_{a,b}$ which we can plug in any optimizing routine to find what parameters (a, b) minimize the absolute simulated error ϵ

A simple example would be fitting the European Central Bank deposit rate as if set by a PI controller trying to keep unemployment rate at 8%. The error time series e_t is then the difference between unemployment rate and 8% while the policy time series p_t is the deposit rate. The result is shown in figure 25. The PI controller has a strong proportional component and a weak integral. This matches the Taylor rule approach which is fundamentally P only.

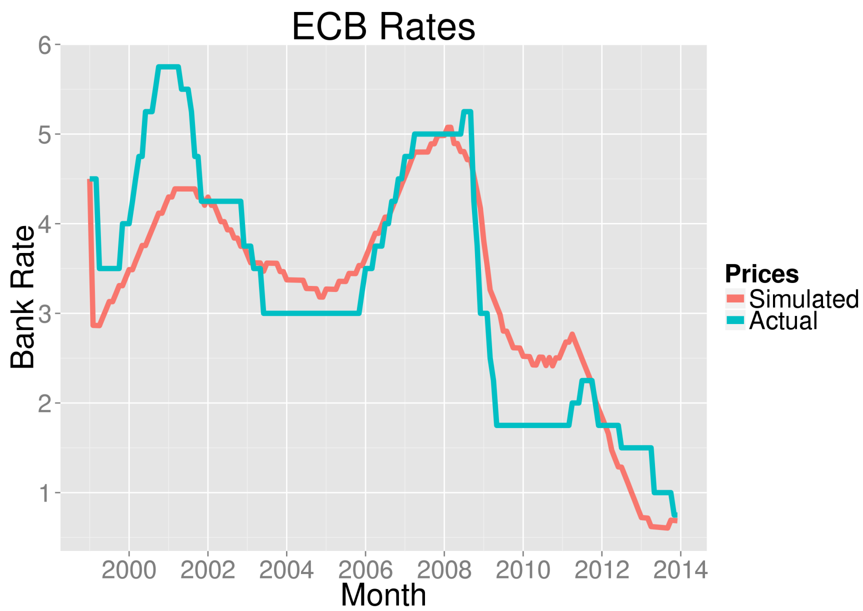

Figure 25: A comparison between ECB rates and the rate simulated by a PI controller targeting unemployment at 8%. The parameters are $a = 0.94$ and $b = 0.0005$

In reality central banks target both unemployment and inflation at the same time. Hawkins, Speakes, and Hamilton (2014) fits a PI controller targeting both (using potential output gap rather than unemployment) to the US central bank and compares the fit favorably with traditional Taylor rules estimations.

For the fit to be informative the data needs to be of high quality. If the time density is too coarse or we use aggregate market data the fit will be meaningless. As an example, imagine a PI controller setting rents in the United States. Take as error time series e_t the monthly rental vacancy rate and as price series p_t the real rent rate (more precisely the urban rent CPI divided by the general CPI). I show the fit in figure 26

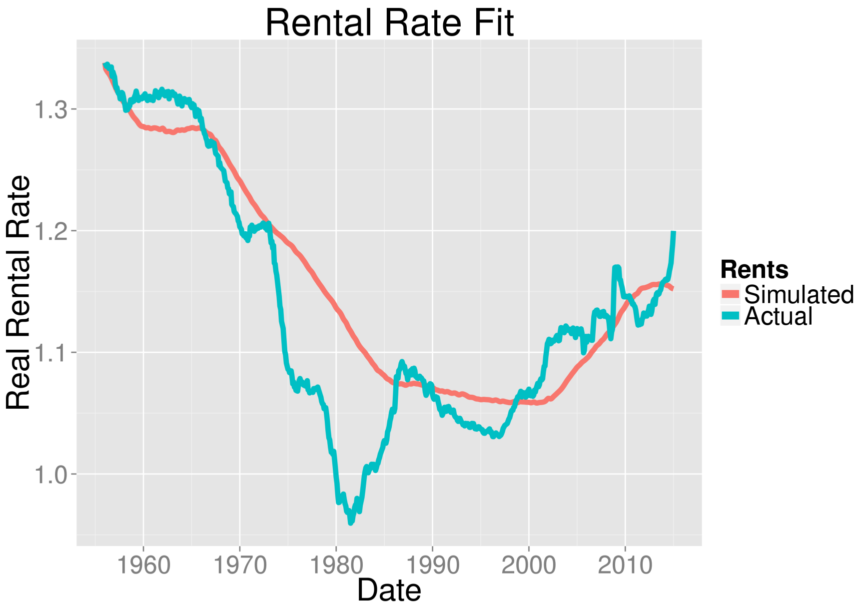

Figure 26: The real urban rental rate in the US and the closest PI output when targeting 8% vacancy rate. Notice that the PI parameters are $a = -0.0013$ and $b = -0.00038$, that is rent goes up when vacancy rate goes up

While the fit might look acceptable the PI parameters are negative. This means that the best PI fit has rent go up when vacancy are high. Rather than capturing owners lowering rent to deal with vacancies I am capturing new rooms being made available when rents are high enough. I replicated the circular causality problem of section 7.3 where over large periods of time (in this case months) two processes occur: prices decline when demand is low but eventually production also declines because of low prices. For the housing market while it is possible that rents decline while vacancies are high it is also true there will be less houses on the market while rents are low. The overall correlation between monthly rent and monthly vacancies could then be of either sign.

What is needed is firm-level, high frequency data. A data that is firm-level but is not high frequency nor has enough observation is the free supermarket data from Aguirregabiria (1999). There are 529 goods, each with 29 observations, one per month. The problem with non central banks data is to figure

Table 2: 10 best PI Fits for the Wholesale Price

dart	aParam	bParam	margin
SIRO MARIA 400 GRS.	-0.000708	0.005561	34.68966
AGUA SOLARES 250 C.C PAK 6	0.000865	0.000000	74.72056
PERA HERO ALMIBAR 450 GRS.	-0.011320	0.022337	90.79018
VINO OLARRA OTOAL 87 ROSADO	-0.002209	0.014678	93.63590
BAYETA VILEDA SUELOS	-0.001862	0.002559	96.65143
PAN LU DE PUEBLO 24 REBANAD.	-0.010047	0.010692	115.45256
LENTEJA ASTURIANA PARDINA KG	-0.001683	0.003155	120.79976
PIMI. VELA PICO LTA. 175 GRS	-0.000475	0.002262	121.79440
FONTANEDA EL COLEGIAL 800 GR	-0.023865	0.039007	124.33759
BOLSA BASURA 25 UND. RF.1007	0.000013	0.000000	126.79908

out what the PI target and the errors are. This is particularly complicated in this data-set because of the presence of inventories, returns and the lack of information on the manufacturers themselves. For this data-set, I use as PI policy p_t the wholesale price, and as PI error the difference between orders placed by retailers to the wholesaler and the orders placed by the wholesaler to the manufacturers. This is sub-optimal because it ignores customer returns and inventory targets, but it is a simple proxy for what must be the sales targets.

Table 2 shows the 10 best PI fits. In some cases, the P parameter is negative, but it is always the case a is smaller than the b parameter so that there is the PI controller never operates in 'reverse'. The median simulated error ϵ is 416.48 Pt, so wrong by about 14 Pt a month. An example of the sucessful fit ($\epsilon \approx 200$) is shown in figure 27.

The main weakness of my estimations is that the simulated estimated error ϵ is computed over training data. A better approach, especially when comparing different model fits, would be to compute ϵ by cross-validation or training data. This was not feasible for these examples because the testing data of the European Central Bank would have been the rates during the crisis which are set more aggressively than the training data would suggest while the wholesale data is too small to afford it being cut into training and testing.

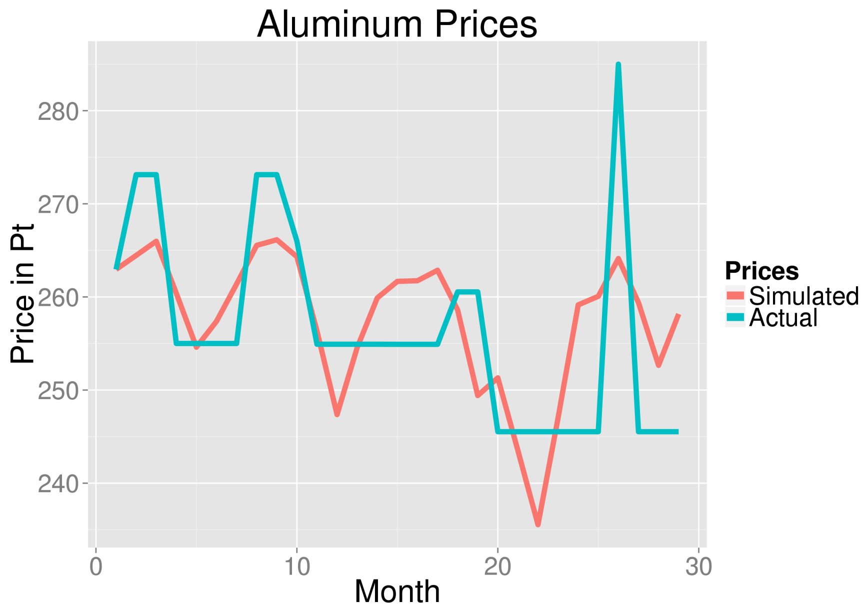

Figure 27: The comparison between PI controller and actual wholesale prices of 30 meters of Reynolds aluminum foil

10 Conclusion

In this paper I showed how trial and error pricing creates bullwhip effects and how sticky prices can fix them. This allowed agents that are too simple to centralize information to coordinate over market prices. I also showed how the result is robust to market structure and knowledge assumptions about price impacts μ and price stickiness s. I believe this paper represents an example of how focusing on interactions and agent-based models can provide new answers and hypotheses to old questions. It is a methodology that allows for expressing and examining time and trading rules' minutiae easily.

There are two paths I can take with this model. The first is improving its overall realism, probably by adding more feedforward elements. Agents here are purposefully simple as a way to show how little is required for markets to coordinate. A better agent would use more data: here agents weren't allowed to even look at competitors' prices. If they use more data artifacts such as those described in section 4.2 would disappear as firms would be on the same level as their consumers. A better agent would also be able to auto-tune its PI parameters during the simulation. I did tune stickiness s in section 4.2 but the process could be generalized to a and b as well.

The second path I can take with this model is to use it as it is with different market structures. I have shown that the Zero-Knowledge firm works well in a monopolist environment because trial and error is at its most informative when the agent is alone. Viceversa I have shown that in a competitive environment trial and error is at its least informative because of the noise generated by the competition and the ease of the customer base to switch providers. I think the Zero-Knowledge agent would thrive in a point between these two extremes. I believe that monopolistic competition, a market where each agent has only partial market power and information is too dispersed for game theory to apply, is the obvious next step.

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